

Constraining Cosmic Expansion and Gravity with Galaxy Redshift Surveys

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We analyze the science reach of a next generation galaxy redshift survey such as BigBOSS to fit simultaneously for time varying dark energy equation of state and time- and scale-dependent gravity. The simultaneous fit avoids potential bias from assuming Λ CDM expansion or general relativity and leads to only modest degradation in constraints. Galaxy bias, fit freely in redshift bins, is self calibrated by spectroscopic measurements of redshift space distortions and causes little impact. The combination of galaxy redshift, cosmic microwave background, and supernova distance data can deliver 5-10% constraints on 6 model independent modified gravity quantities.

I. INTRODUCTION

The power spectrum of matter density perturbations contains information on both the cosmic expansion history and growth history. Comparison of these fundamental evolutions not only tightens constraints on the cosmological model but enables tests of the framework, such as the validity of general relativity as the theory of gravity. However, most cosmological parameter estimation, from either current or projected data, either assumes general relativity, and hence fixes the growth history to be determined by the expansion history, or assumes a Λ CDM expansion history to test gravity.

If either assumption is incorrect then the result will be biased. Thus, even if one is only interested in constraining the expansion history and dark energy properties, for example, one still must fit for gravity in order to obtain robust results for the expansion. Current data cannot tightly constrain either quantity without ad hoc assumptions such as no time dependence. The volume and redshift reach of cosmological surveys is rapidly improving, however, and projections for the next generation are for the $\sim 10\%$ accuracy level on the time variation of dark energy properties or gravity.

The question we address is how the constraints are affected when the matter power spectrum data measured by galaxy redshift surveys are fit simultaneously for varying dark energy equation of state, scale- and time-dependent gravity, and astrophysical evolution such as galaxy bias. Aspects of this have been considered in the literature with some restrictions: for example [1] parametrized gravity with the scale- and time-independent gravitational growth index γ and a single bias parameter, while [2] extended this to binned bias; [3, 4] assumed scale-independent gravity with a particular time variation but allows scale- and time-dependent bias; [5] applied a motivated scale- and time-dependent gravity parametrization to current data, restricting to quantities independent of galaxy bias. (Note that more general simultaneous fitting, such as with principal component analysis, has been applied to other cosmological probes such as weak lensing, e.g. [6, 7].)

With the aim of deriving more general results that avoid assuming a particular model, for expansion history, gravity, or astrophysical bias, we use scale- and time-dependent bins of gravity and time varying effective dark energy equation of state and bins of galaxy bias. Specifically, we fit for gravitational modifications to the two Poisson equations (for the behavior of matter and of light) as freely floating values at high and low redshift and large and small scales, i.e. in bins of redshift z and wavenumber k . Galaxy bias is fit as independent values in 17 bins of redshift. Dark energy evolution is treated through the highly accurate w_0 - w_a parametrization. Due to the presence of non-Gaussian covariances between the many parameters we carry out a Markov Chain Monte Carlo exploration of parameter space using simulated next generation cosmological survey data.

In Sec. II we explain the treatment of dark energy, gravity, and galaxy bias in detail. The simulated data sets used are presented in Sec. III. We analyze the results, with particular attention to covariances and the effect on constraints of not assuming fixed expansion, fixed gravity, or fixed galaxy bias in Sec. IV. Implications for the science reach of next generation surveys are discussed in Sec. V.

II. DARK ENERGY, GRAVITY, AND GALAXIES

Matter density perturbations grow under gravitational instability but at a rate suppressed by cosmic expansion. Therefore the evolution of large scale structure clustering depends on the matter density Ω_m , the expansion rate H , and the laws of gravity. In addition, galaxy redshift surveys do not directly measure the mass power spectrum in real space but the galaxy clustering in redshift space. This introduces two additional ingredients: the statistical distribution of galaxies may be biased relative to the mass (i.e. dark matter), and the velocity field of the galaxies, generated by the gravitational potentials of the structures, adds anisotropic distortions to statistically isotropic density field.

The redshift space distortion depends on the angle with respect to the line of sight and its overall amplitude is determined by the growth rate of structure at the given redshift, $f(z)$. This carries with it important additional cosmological (and gravitational) information, enhancing the reach of spectroscopic galaxy surveys. Galaxy bias b , however, has the potential to confuse extraction of the total amplitude of growth up to the given redshift. Since it enters without angular dependence, one can fit separately the two effects, although each is convolved with the overall mass fluctuation amplitude often denoted by $\sigma_8(z)$. That is, cosmological information comes in the form of $b\sigma_8(z)$ and $f\sigma_8(z)$ [8]. The amplitude $\sigma_8(z)$ is proportional to the linear growth factor $D(z)$, which is related to the gravitational potential through a Poisson equation, and so this growth history depends on both the expansion history and gravity. If additional data directly related to the mass fluctuations (rather than galaxies) is available, for example through weak gravitational lensing, then $D(z)$ can be separately determined, although lensing also depends on gravity, in its own way.

Thus to draw general robust conclusions on expansion and dark energy properties, for example, we should fit not only for $H(z)$ but simultaneously for gravity – the two histories then determining $D(z)$ and $f(z) = -d\ln D/d\ln(1+z)$ – and galaxy bias. Moreover, the way we treat each of these quantities should be valid over a wide range of cosmologies so that the results are not overly model dependent.

For the expansion history we work within the flat Friedmann-Robertson-Walker framework, where the expansion rate is described by the matter density Ω_m as a fraction of the critical density and the time dependent dark energy equation of state $w(z) = w_0 + w_a z/(1+z)$. This form for $w(z)$, obtained from exact solutions for a wide range of dark energy and gravity models [9], has been demonstrated as a calibration relation with an accuracy of 0.1% for observables [10]. Perturbations in the effective dark energy (i.e. even if there is no physical dark energy, just a modification of gravity) are treated consistently within the equations of motion.

Note that while many papers take an exact Λ CDM expansion history when allowing for modified gravity, this is highly model dependent. While simple $f(R)$ gravity models can be treated in terms of the scalaron mass, which determines both the growth and expansion behaviors so that effective screening within the solar system (really, small df/dR) requires negligible deviations of $w(z)$ from -1 , this is not generally true. For example, DGP gravity [11] has strong deviations in both growth and expansion from Λ CDM, though as it is a one parameter model they are tied together. Galileon models, however, have sufficient freedom that the significant deviations in expansion history can appear alongside moderate growth deviations (see, e.g., [12]). Thus fitting for expansion and gravity independently is most generic and free from possibly unwarranted assumptions.

To account for possible extensions beyond general rel-

ativity, and their effects on matter growth and lensing of light, we solve for the two metric potentials ϕ and ψ in modified Poisson equations

$$\nabla^2\psi = 4\pi G_N a^2 \delta\rho \times G_{\text{matter}} \quad (1)$$

$$\nabla^2(\phi + \psi) = 8\pi G_N a^2 \delta\rho \times G_{\text{light}}. \quad (2)$$

The rigor and completeness of these equations, together with those for the matter density and velocity from energy-momentum conservation, is discussed in detail in, e.g., [13–16] (though here we have simplified the notation) and clearly related to the nonrelativistic and relativistic geodesic equations in [17].

Parametrization of the generally time- and scale-dependent functions $G_{\text{matter}}(k, z)$ and $G_{\text{light}}(k, z)$ can be specialized to forms expected from scalar-tensor or DGP gravity, for example (see [5] for a unification of the two), or kept free form through a principal component analysis [6, 7, 15, 18], say. Since we wish to carry out a substantially model independent analysis, but be able to interpret clearly physically the constraints on gravity, we follow the high vs. low redshift, large vs. small scale binning approach of [13, 16]. This allows for 8 free gravity parameters: G_{matter} and G_{light} values in bins from $z = 0-1$ and $z = 1-2$ (for $z > 2$ their values are set to 1, i.e. behaving as general relativity in the high redshift, high curvature universe), and in wavenumber bins $10^{-4}\text{Mpc}^{-1} < k < 10^{-2}\text{Mpc}^{-1}$ and $0.01\text{Mpc}^{-1} < k < 0.1\text{Mpc}^{-1}$. Because of the uncertainty in the appropriate growth equations in the nonlinear density region at higher wavenumbers for an arbitrary gravity theory (e.g. the presence of various screening mechanisms [19]), and in the velocity induced redshift space distortions to the galaxy power spectrum, we conservatively do not use data at $k > 0.1\text{Mpc}^{-1}$.

Galaxy bias is treated through 17 free parameters representing the values $b(z)$ in bins of width 0.1 in redshift from $z = 0.1-1.8$. Since we only include large scale information we take the bias to be scale independent. We phrase the bias amplitude in terms of the comoving clustering expectation, $b(z) = b_0(z) D(z=0)/D(z)$ and fit for b_0 in each redshift bin. We use independent sets $\{b_{\text{LRG}}(z_i)\}$ and $\{b_{\text{ELG}}(z_i)\}$ for $b_0(z)$ for luminous red galaxies and emission line galaxies in the survey.

In addition to these parameters we also fit for the physical baryon density $\Omega_b h^2$, physical matter density $\Omega_m h^2$, ratio of cosmic microwave background (CMB) sound horizon to angular diameter distance to last scattering θ , optical depth to reionization τ , scalar spectral tilt n_s , amplitude of primordial scalar perturbations A_s , and any astrophysical nuisance parameters appropriate to the data sets.

III. GALAXY REDSHIFT SURVEY SCIENCE

Spectroscopic galaxy surveys provide three dimensional information on galaxy clustering and direct measurements of the galaxy density and velocity fields, thus probing the growth history. BigBOSS [20] is planned to

survey $50 (\text{Gpc}/h)^3$ of cosmic volume, obtaining redshifts of 20 million galaxies from $z \approx 0.5$ –1.8. (In addition, it provides data on quasar clustering and on neutral hydrogen density fluctuations through the Lyman alpha forest, ranging over $z = 1.8$ –3.5, but conservatively we do not include constraints from those probes.)

The redshift space galaxy power spectrum to be compared with observations is related to the linear theory mass power spectrum calculated from the equations of motion (Poisson equations and energy-momentum equations) by

$$P_g^z(\mathbf{k}, z) = (b + f\mu^2)^2 P_{m,lin}^r(k, z), \quad (3)$$

where the squared factor is the Kaiser correction [21]. This is a good approximation strictly in the linear regime; since we use only $k < 0.1 \text{ Mpc}^{-1}$ this is not too unreasonable. How to map the isotropic mass power spectrum to the anisotropic galaxy power spectrum is not settled beyond linear theory so we do not attempt to go beyond the Kaiser formula here. (See [2, 22] for explorations of beyond linear theory constraints on the scale independent gravity parameter γ as a function of maximum wavenumber, and [23] for specific $f(R)$ gravity models.)

Uncertainties on the measurement of the galaxy power spectrum depend on the volume surveyed in each redshift shell and the number density n of each type of galaxies used to sample the density field. The volume is determined by the area of sky (14000 deg^2 for BigBOSS) and the cosmological distance-redshift relation. We use the redshift distributions $n(z)$ for the two types of galaxies given in [20]. See [2, 24, 25] for further details on calculating the likelihood with galaxy power spectrum measurements.

In addition, we include simulated Planck CMB data. This helps to constrain the cosmological parameters, especially the primordial ones, and the gravitational modifications through the integrated Sachs-Wolfe effect and CMB lensing. The ISW effect has sensitivity to G_{light} since this Poisson equation governs the behavior of relativistic geodesics; CMB lensing is sensitive to both G_{matter} through growth in the matter power spectrum and G_{light} through light deflection. We include CMB lensing through the deflection angle power spectrum as prescribed by [26]. (See [27] for its use in constraints on the gravity parameter γ .) For constraints on the expansion history we employ simulated supernova distance-redshift measurements of 1800 supernovae over $z = 0$ –1.5 with a systematic floor of $0.02(1+z)/2.5$ magnitudes, and include the supernova absolute magnitude parameter \mathcal{M} as a nuisance parameter.

IV. CONSTRAINING EXPANSION AND GRAVITY SIMULTANEOUSLY

We carry out a Markov Chain Monte Carlo analysis to find the joint posterior likelihoods of the parameters, with our modified versions of CAMB [28] and CosmoMC

[29]. Our baseline analysis consists of projected galaxy clustering, CMB, and supernova data, allowing for independent fits of the 8 binned gravity parameters G_{light} and G_{matter} , 27 binned galaxy bias parameters for LRG and ELG, the dark energy equation of state parameters w_0 , w_a , plus the other cosmological and nuisance parameters. To speed the convergence of our Markov Chain Monte Carlo implementation, we fit for the maximum-likelihood bias combination at each step in cosmological parameter space. Because of this, our quoted credibility regions should be considered conservative, as outlying points in cosmological parameter space are assigned a larger posterior density than they might be if we marginalized over the bias parameters. We then consider the following variations to explore the impacts of physical ingredients:

- Expansion: fix to Λ CDM background, i.e. $w_0 = -1$, $w_a = 0$
- Gravity: fix to general relativity, i.e. $G_{\text{matter}} = 1$, $G_{\text{light}} = 1$ for all bins
- Galaxy bias: fix to inverse growth, but fit for constant b_0^{LRG} , b_0^{ELG}

We emphasize that our baseline case varies all these ingredients.

The results for the gravitational sector in the baseline case and its physical variations are shown in Figs. 1. We see that for next generation data sets there is little degradation in fitting for dark energy equation of state and galaxy bias simultaneously with gravity, giving great promise to our ability to test the cosmological framework. Adding parameters for non- Λ CDM expansion has only a percent level effect (except $\sim 15\%$ on G_{matter} at large k and z), while bias parameters only affect G_{matter} at large k and only loosen the constraints by at most $\sim 50\%$.

The quantity G_{light} will be determined to $\sim 5\%$ (at 68% CL) independently at low and high redshift, and small and large wavenumber, while G_{matter} will be tested to $\sim 8\%$ on small scales ($k > 0.01 \text{ Mpc}^{-1}$) where the galaxy survey data is most important, and to $\sim 35\%$ on large scales ($k < 0.01 \text{ Mpc}^{-1}$). The much weaker constraints at large scales, where we assume no galaxy data, demonstrate the importance of the galaxy power spectrum measurement in constraining gravity. Even so, the constraints on large scale G_{matter} show more than a factor of two improvement over the yellow shaded contours in Figure 11 of [16] (their \mathcal{V} is our G_{matter}), due to our inclusion of CMB lensing.

The gravity parameters have little covariance with each other (including between large and small scale bins of the same parameter, not shown), except mildly at low redshift. Excellent complementarity exists between the quantities and the probes: galaxy data constrains G_{matter} , CMB data predominantly constrains G_{light} (again see Figure 11 of [16] and Figure 7 of [13]), and supernova data constrains the expansion dynamics of w_0 , w_a , ignoring the gravity parameters.

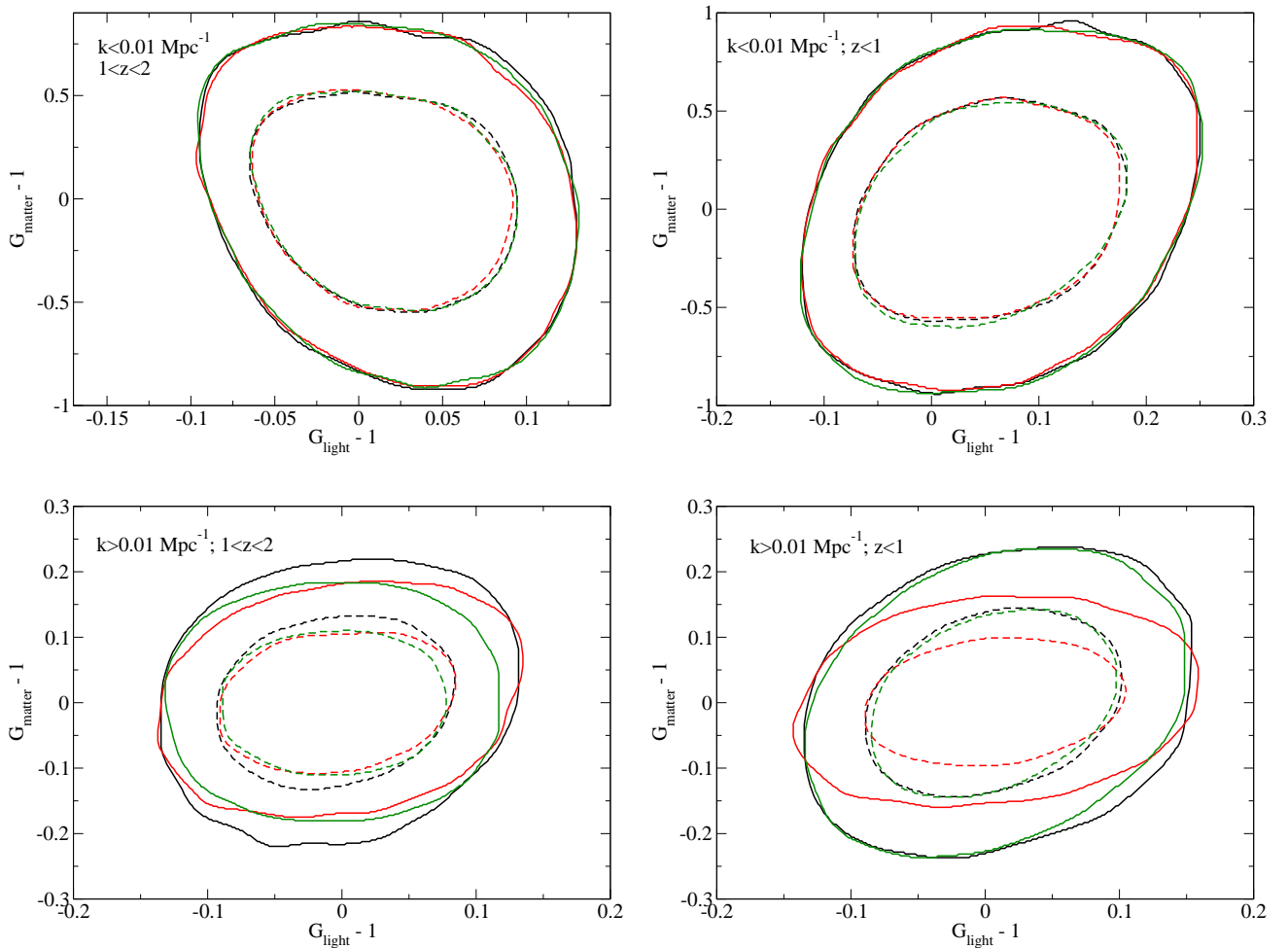


FIG. 1. 68% and 95% projected confidence limit contours for the gravitational parameters binned in low and high redshift, low and high k are plotted for the baseline case of fitting for gravity, expansion, and galaxy bias (black curves), and the variations of fixing expansion by $w = -1$ (light green curves) or fixing galaxy bias to inverse growth (dark red curves).

The expansion sector is also not strongly affected by the different cases, when next generation data is available. Figure 2 shows the contours in the baseline case, when fixing galaxy bias, and when fixing to general relativity. The w_0 - w_a contour increases in area by $\sim 30\%$ when including the fit for the 8 gravity parameters, with $\sim 2\%$ loosening in w_0 and $\sim 20\%$ in w_a determination. As expected from these and the previous results, plots of the crosscorrelations of gravity and expansion parameters (not shown) display little covariance, with only a minor correlation between w_a and G_{matter} at large k .

V. CONCLUSIONS

Testing gravity on cosmic scales is crucial to understanding acceleration and fundamental laws of physics. The inability of present data sets to significantly constrain the laws of gravity affecting matter (i.e. G_{matter})

is the largest obstacle to efforts to test general relativity on cosmological scales and to constrain modified gravity theories. Next generation astronomical surveys propose to remove this obstacle by presenting us with detailed redshift-space maps of the distributions of galaxies throughout the Universe.

We have shown that these maps will, indeed, be able to deliver on their promise to constrain gravity at cosmological scales, even given our imperfect knowledge of both the cosmic expansion history and the bias factor relating galaxy to dark matter clustering. While a theoretical framework predicting the galaxy-dark matter bias would certainly be welcome, the results show it is not requisite for cosmological tests of gravity. Physics such as a generalized expansion history and astrophysics such as galaxy bias should be included in the fits (rather than assumed known) to avoid erroneous conclusions about gravity, and can be included without fear of degrading observational constraints beyond usefulness. Conversely, future exper-

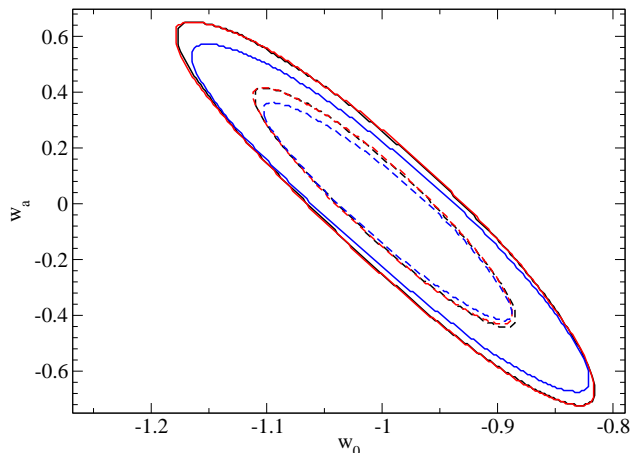


FIG. 2. 68% and 95% confidence limit contours for the effective dark energy equation of state parameters w_0 and w_a are plotted for the baseline case of fitting for gravity, expansion, and galaxy bias (black curves), and the variations of fixing to general relativity (light blue curves) or fixing galaxy bias to inverse growth (dark red curves).

iments can proceed without fear of their science output being obscured by our ignorance.

The model independent approach to testing gravity

used here avoids restriction to a specific model, and has excellent orthogonality among its quantities and between its quantities and the expansion and galaxy bias parameters. Strong complementarity exists as well between the cosmic probes: galaxy surveys testing G_{matter} , CMB measurements constraining G_{light} , and supernova distances measuring expansion. Together, the next generation combination of these data will deliver 5–10% measurements of 6 gravity quantities, plus 35% measures of the remaining 2. These can potentially be improved further by going beyond the purely linear regime (we conservatively cut off $k > 0.1 \text{ Mpc}^{-1}$) once modified gravity effects on the redshift-space galaxy power spectrum are better understood, by higher quality CMB lensing from ground based polarization experiments, or by inclusion of crosscorrelations between galaxy maps and the CMB.

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